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5. AUTHOR(S)

DR. LUTHER WHITE

2304/A1  
AFOSR-91-0017  
61102f

UNIVERSITY OF OKLAHOMA  
DEPARTMENT OF MATHEMATICS  
NORMAN, OKLAHOMA 73019

AFOSR-TR- 94 0211

AFOSR/NMIS MONITORING AGENCY NAME(S) AND ADDRESS

110 DUNCAN AVE, SUITE B115  
BOLLING AFB DC 20332-0001

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This project seeks to study the estimation of elastic, damping, and material parameters inflexible structure. Of particular interest are problems in the design and estimation of parameters in structures made up of systems of coupled beams and plates, the estimation of parameters in models that may not have unique solutions, and the estimation and design of various plate and shell models incorporating, for example, large deformation, variable thickness, existing curvatures, contact and possible friction conditions.

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in Linear and Nonlinear Distributed  
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Principal Investigator: Professor Luther W. White  
Department of Mathematics  
University of Oklahoma  
Norman, Oklahoma 73019

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# Estimation and Control of Parameters in Linear and Nonlinear Distributed Models of Flexible Structures

Final Report

February, 1994

## 1. Introduction and Goals.

This project seeks to study the estimation of elastic, damping, and material parameters in flexible structures. Of particular interest are problems in the design and estimation of parameters in structures made up of systems of coupled beams and plates, the estimation of parameters in models that may not have unique solutions, and the estimation and design of various plate and shell models incorporating, for example, large deformations, variable thickness, existing curvatures, contact and possibly friction conditions.

Because of their application as constructional elements, it seems important to study both the static and dynamic behavior of beams, plates, and shells, as well as their combinations. Accordingly, this project seeks to study the identification of parameters and the design of structures modeled by distributed systems. Such efforts are fundamental to our ability to simulate and control their behavior. Our goals are (i) develop and test numerical algorithms for the estimation of parameters and design of certain structures composed of connected beams and plates, (ii) develop theories and test numerical algorithms for the estimation of parameters in models that may have multiple states, (iii) use the above theories to consider various models of plates, shells, and beams with possible nonlinearities, and in cases where feasible (iv) test algorithms with laboratory data when possible.

An identification or estimation problem seeks to determine parameters within a mathematical model from observed data. The central issue is how to utilize the data to determine the desired parameters within the context of the model. A design problem views parameters within the problem as controls to be adjusted so as to produce desired results under certain conditions. Although these problems are stated differently, a common mathematical optimization formulation may be given to them. Available is a model equation

$$(1.1) \quad \mathcal{E}(q) \quad u(q) = f$$

in which the parameters  $q$  to be estimated belong to a specified admissible set  $Q_{ad}$  of a Banach space  $Q$  and solutions  $u = u(q)$  belong to a Banach space  $X$  of states. The set  $Q_{ad}$  should be physically meaningful. However, whereas in previous work it has been a condition that the parameter-to-state mapping is well-defined as a function from  $Q_{ad}$  into the state space  $X$  we will in this work allow the possibility of multiple solutions for the state-equation.

Available (for the estimation problem) are data  $z$  that we view as belonging to an observation space  $Z$ . An approach to solving the estimation problem is formulated as a minimization problem:

Find  $q_0 \in Q_{ad}$  such that

$$(1.2) \quad J(q_0, z) = \inf\{J(q, z) : q \in Q_{ad}\}$$

admissible functions. An example, if sense can be given to it, of a fit-to-data functional  $J(q, z)$  is the so called regularized output least squares functional

$$(1.3) \quad J(q, z) = \|Cu(q) - z\|_Z^2 + \rho \|q\|_Q^2$$

where  $Z$  is the observation space (Hilbert space),  $C$  is the observation operator that takes  $X$  into  $Z$ , and  $\rho > 0$  is a regularization parameter. If there may be multiple solutions to (1.1), care must be given in the formulation of the fit-to-data functional such that it is a well-defined functional that is bounded below, is weakly-lower semicontinuous over  $Q_{ad}$  with respect to a topology in which  $Q_{ad}$  has suitable compactness properties. There are other approaches to identification but these typically assume the parameter-to-state mapping is well-defined and we refer the reader to [1] for discussions. Issues of interest for (1.1)-(1.2) include the proper formulation, regularity properties of solutions of (1.2), approximation, stability with respect to the data, and uniqueness or identifiability.

The minimization problem (1.2) may also be viewed as design problem where  $z$  represents a target function that we wish to reach by choosing a suitable parameter  $q$ . Certainly, it makes sense from a design point of view to impose as a condition of the design problem that the mapping  $q \mapsto u(q)$  is well-defined, although one does not have to. Design problems seem to be less data oriented

than estimation problems since one can often exercise more control over the formulation. In estimation problems one is limited by the ability to measure the state. The data are not always in the most desirable form. Indeed this is one of the first lessons that one learns attempting to use experimental data.

If one expects to use estimation techniques to identify parameters from measurements, then one must first decide upon the a mathematical model (1.1). Hence, one must decide whether the mathematical model is appropriate for the physical system being observed. The mathematical model embodies the pertinent principles and assumptions based on physics and continuum mechanics. Certainly, one must be cautious about using models to fit data that do not satisfy the assumptions upon which those models are derived. It is always desirable to validate in some way the model in question. For example, comparing spectral data predicted by the model with spectral data observed in the laboratory provides a reasonable step in model comparison.

Having proposed the mathematical model, one must address how to use the data within the framework it imposes for the purpose of estimation and mode validation. The data for systems of interest in this project are obtained as "pointwise" measurements of deformation or strain in the static case or acceleration or velocity or their Fourier transforms at points in the dynamic case. These data should be in a form consistent with the mathematical model having the proper units, etc. The mathematical analysis of identification problems such as (1.1)-(1.2) gives information on how best to formulate estimation problems to use these data.

In the following we indicate some of the results we have obtained during the last year. In Section 2 we discuss the estimation of elastic parameters in dynamic von Karman plate models. In Section 3 we report on our work with Prof. D. L. Russell on the modeling and spectral identification for so called narrow plate models that should be useful in modeling wings, fan blades, and corrugated structures. Finally, in Section 4 we indicate our work on the modeling and estimation of parameters in dynamic models of coupled-beam systems.

## 2. Estimation of elastic parameters in a dynamic nonlinear plate model.

During the funded period another ther area of investigation

has been the estimation of parameters in static and dynamic versions of plate models that allow large deformations. For the dynamic problems we have considered the von Karman and Morozov models. It is well-known that global solutions of the von Karman equations are, in general, nonunique [2] although local uniqueness of classical solutions for certain cases has been demonstrated in [3,4]. In order to give a mathematical formulation of the parameter identification problem for either the static or dynamic von Karman equations, it is necessary to study the estimation of parameters for systems that may not have well-defined parameter-to-state mappings. We thus consider a theory for the estimation of parameters for set-valued parameter-to-state mappings.

We distinguish between two cases static and dynamic. For the static problem it is apparent the the Galerkin systems may not have unique solutions. Our approach is to introduce model error equations. See [8] and the description of the model error method in our original proposal. One advantage that we have discovered with the model error method in the static case is that we may give formulations to avoid having to calculate the adjoint completely and may be able to compute the derivatives of the fit-to-data functional directly from the state solutions. This obviously can lead to significant computational savings. Alternatively, for the static case we may seek conditions that imply that the parameter-to-state mapping is well-defined. In [9] this approach is taken that is based on the contraction mapping principle for existence as opposed to the Schauder fixed point principle [2].

In the dynamic case we can take advantage of fact that the Galerkin system forms a system of ordinary differential equations for which we may obtain the existence of unique solutions. Our approach is first to formulate and analyze the parameter identification problem for the von Karman problem using an existence theory for set-valued parameter-to-state mappings. A formulation is given that utilizes the weak\* compactness of the solution set for admissible parameters. There are complicating technical factors, however, that imply different convergence results [10,11]. We show existence of a solution to the output-least-squares estimation problem. Further, we obtain certain weak stability results and convergence properties for the estimation problems of associated Galerkin systems. Next we use the fact that the Galerkin system possesses a parameter-to-state mapping that

is well-defined to pose well-defined output-least-squares estimation problems. We then determine regularity and approximation results for the optimal estimators of the Galerkin problems and use them to prove stability and convergence results for the estimation problems for which the Galerkin systems are the underlying systems.

We give the estimation problem for the Galerkin problem and report the results of several numerical experiments. The results of other numerical studies as well as the theoretical development are reported in references [10] and [11]. We recall the von Karman system

$$(2.1)(i) \quad u_{tt} + \Delta (a \Delta u) + \epsilon [B(u,u), u] = f \text{ in } Q$$

with initial conditions

$$(2.1)(ii) \quad \begin{aligned} u(0) &= u_0 \\ u_t(0) &= u_1 \end{aligned}$$

and boundary conditions

$$(2.1)(iii) \quad u = \frac{\partial u}{\partial n} = 0 \text{ on } \partial Q \times (0, T)$$

where

$$(2.1)(iv) \quad \Delta^2 B(u, v) = [u, v]$$

with clamped boundary conditions. Let  $\{\varphi_k: k=1, \dots, N\}$  be linearly independent subset of functions in  $V$  and  $\{\psi_k: k=1, \dots, M\}$  be a linearly independent subset of functions in  $Q$ . We denote by  $V^N = \text{span}\{\varphi_k: k=1, \dots, N\}$  and  $Q^M = \text{span}\{\psi_k: k=1, \dots, M\}$ . We set  $u^N = \sum_{k=1}^N c_k(t) \varphi_k$  and  $a^M = \sum_{i=1}^M a_i \psi_i$ . Defining the matrices

$$(G_0)_{ij} = \int_{\Omega} \varphi_i \varphi_j \, dx \text{ and } (G_2)_{ij} = \int_{\Omega} \Delta \varphi_i \Delta \varphi_j \, dx$$

$$(G^k)_{ij} = \int_{\Omega} \psi_k \Delta \varphi_i \Delta \varphi_j \, dx \text{ for } k=1, \dots, M$$

and

$$(H^k)_{ij} = \int_{\Omega} [\varphi_i, \varphi_j] \varphi_k \, dx \text{ for } k=1, \dots, N,$$

we set

$$H(c, d) = \begin{bmatrix} c^* & \vdots & H^k & d \end{bmatrix} = \begin{bmatrix} d^* & \vdots & H^k & c \end{bmatrix} = \mathcal{H}(d) c$$

Finally, we define the  $N \times N$  matrix



$$G(a) = \sum_{k=1}^M a_k G^k$$

The following semi-discrete system is obtained

$$\begin{aligned} G_0 c_{tt} + G(a) c + \epsilon H(c, d) &= f \\ (2.2) \quad G_2 d &= H(c, c) \\ G_0 c(0) &= \mu_0 \\ G_0 c_t(0) &= \mu_1. \end{aligned}$$

We use the following difference equations for a time approximation

$$\begin{aligned} G_0 \frac{c_{i+1} - 2c_i + c_{i-1}}{h^2} + G(a) \frac{c_{i+1} + 2c_i + c_{i-1}}{4} + \\ + \epsilon H(c_i, d_i) = f_i \end{aligned}$$

where  $h$  denotes the time difference and the subscript  $i$  indicates the time  $t_i = i * h$ . Thus, the following sequence is generated

$$\begin{aligned} G_0 c_0 &= \mu_0 \\ G_0 c_1 &= \mu_0 + h \mu_1 \\ G_2 d_i &= H(c_i, c_i) \\ (G_0 + (\frac{h}{2})^2 G(a)) c_{i+1} &= 2 (G_0 - \frac{h^2}{4} G(a)) c_i - \\ - (G_0 + (\frac{h}{2})^2 G(a)) c_{i-1} &+ h^2 (f_i - \epsilon H(c_i, d_i)) \end{aligned}$$

The derivative system is obtained by formally differentiating the equations in (2.1) with respect to  $a_p$  for  $p = 1, \dots, M$ . Thus, we see that the partial derivative of  $c$  and with respect to  $a_p$ ,  $D_p c$  and  $D_p d$ , satisfy the system

$$\begin{aligned} G_0 (D_p c)'' + (G(a) + \epsilon H(d)) (D_p c) + \\ + \epsilon H(c) D_p d = - G^p (\delta c' + c) \\ (2.3) \quad G_2 (D_p d) = 2 H(c) (D_p c) \end{aligned}$$

with initial conditions given by

$$(D_p c)(0) = (D_p c)'(0) = 0$$

To specify the fit-to-data functional, we define the  $M \times M$  matrix  $G^\Psi$  by

$$(G^\Psi)_{ij} = (\psi_i, \psi_j)_V$$

for  $i, j = 1, \dots, M$ , the  $N$ -vector valued function  $t \mapsto z(t)$  by

$$z_i(t) = (\varphi_i, z)_H$$

for  $i=1, \dots, N$ , and the real valued function

$$c_z(t) = \int_{\Omega} z^2(x, t) dx.$$

With these definitions the fit-to-data functional becomes

$$(2.4) \quad J(a) = \int_0^T (c^*(t) G_0 c(t) - 2 z^*(t) c(t) + c_z(t)) dt + \\ + \rho a^* G^\Psi a.$$

Thus,  $J$  is a mapping from  $\mathbb{R}^M$  into  $\mathbb{R}$ . We may calculate the Fréchet partial derivative with respect to  $a_\rho$  to obtain

$$D_{\rho} J(a) = 2 \int_0^T (G_0 c(t) - z(t))^* (D_{\rho} c)(t) dt + 2 \rho (a^* G^\Psi)_{\rho}$$

where  $D_{\rho} c$  is the solution of (2.3).

To solve the estimation problem, we attempt to minimize the functional  $J$ . Our approach is to use a steepest descent method to obtain  $a$ -vectors that decrease the size of  $J(\cdot)$ . We present the results of several numerical experiments. Our computations are conducted for  $\Omega = (0,1) \times (0,1)$  on which we specify a mesh obtained from uniform meshes on  $(0,1)$   $\Delta_1$  with 8 subintervals and  $\Delta_2$  with 3 subintervals. The basis functions  $\{\varphi_i\}_{i=1}^N$  are obtained as tensor products of cubic B-splines over the mesh  $\Delta_1$  that are adjusted to satisfy the clamped essential boundary conditions. Hence, for our examples,  $N = 49$ . On the other hand for the approximation of the parameter, we use cubic B-spline over the mesh  $\Delta_2$  with no restriction on the boundary. The basis functions  $\{\psi_i\}_{i=1}^M$  are obtained as tensor products of these functions. Hence,  $M = 36$ .

We consider the following test problems. Let  $\varepsilon = 0.1$ . Defining the function

$$u_T(x) = 16 x^2 (1 - x)^2,$$

we specify the deformations as

$$u_{tst}(x, y, t) = u_T(x) * u_T(y) * \cos(t) \text{ in } \Omega \times (0, T).$$

Hence, the initial conditions for our examples are given by

$$u_0(x, y) = u_T(x) * u_T(y) \text{ in } \Omega$$

and

$$u_1(x, y) = 0 \text{ in } \Omega.$$

To carryout various experiments to recover the coefficient  $a$ , we specify a particular coefficient  $a_{0tst}$ , then we generate the resulting force vector by using the weak form of the equations (2.1) with basis functions  $\varphi_j$  replacing  $\varphi$ . Thus, from the data  $u_{tst}$  as an observation and  $f$  as an assumed known force, we wish to recover the coefficient  $a$ .

We give the results of several experiments. In all the examples we take the

initial guess for  $a_0(x,y) = 0.5625$ .

In the following we give results in terms of the relative  $L^2$  errors for  $a_0$  at a particular iteration that are calculated as follows

$$\text{Relative } L^2 \text{ error} = \frac{(\int_{\Omega} (a_{0calc} - a_{0tst})^2 dx dy)^{1/2}}{\|a_{0tst}\|_{L^2(\Omega)}}$$

Example 1.

$$a_{0tst}(x,y) = 1.0 + 0.25*\cos(2\pi x)*\cos(2\pi y)$$

iteration	Relative $L^2$ error
0	0.462
1	0.299
5	0.143
10	0.0894
15	0.0684
35	0.0428

Example 2.

$$a_{0tst}(x,y) = 1 + \frac{x^2 + y^2}{2}$$

iteration	Relative $L^2$ error
0	0.610
1	0.410
5	0.213
10	0.141
15	0.0987
35	0.0399

Example 3.

$$a_{0tst} = \begin{cases} 1.5, & x \leq 0.5 \text{ and } y \leq 0.5 \\ 1.0, & \text{otherwise} \end{cases}$$

iteration	Relative $L^2$ error
0	0.550
1	0.363
5	0.180
10	0.112
15	0.0692
35	0.0323

If data are available as a time series, we wish to use data at each time step to estimate the coefficient "a" as the system is evolving in time. That is, at each time step new data is available to use to estimate a. Thus, as the von Karman system is being solved iteratively, we seek to estimate a at the current step with the data using the new information. Similar methods are used in meteorology to incorporate measurements into the solutions calculated by meteorological models and are referred to as forward assimilation techniques. We apply these methods to estimation problems for the case of the von Karman equations.

Specifically we consider the time approximation with a step of length h

$$u_{tt} \rightarrow (u_{k+1} - 2u_k + u_{k-1})/h^2$$

and introduce an error function  $w = w(a, u)$  such that

$$(2.5)(i) \quad \begin{aligned} L w = & u_{k+1} - 2u_k + u_{k-1} + \\ & + h^2(\Delta(a \Delta u_k) + \epsilon [B(u_k, u_k), u_k] - f_k) \text{ in } Q \end{aligned}$$

with initial conditions

$$(2.5)(ii) \quad \begin{aligned} u(0) &= u_0 \\ u_1 &= h u_1 + u_0 \end{aligned}$$

and boundary conditions

$$u_k = \frac{\partial u_k}{\partial n} = 0 \text{ on } \partial\Omega \times (0, T)$$

where  $B(u, v) \in H_0^2(\Omega)$  is the solution of the boundary value problem

$$\Delta^2 B(u, v) = [u, v].$$

We give two approaches to the problem that we consider. The first is to replace the term

$$\Delta (a \Delta u_k) \text{ by } \Delta (a \Delta u_{k+1}),$$

set  $w = 0$ , and consider the elliptic problem

$$u_{k+1} + h^2 (\Delta (a \Delta u_{k+1})) = 2 u_k - u_{k-1} - \varepsilon h^2 ([B(u_k, u_k), u_k] - f)$$

for the estimation of the coefficient  $a$ . This approach would take advantage of the existence and uniqueness theory of local solutions. A second formulation is weaker in the sense that we chose, for example, the operator  $L$  such that  $L = \Delta^2$  with the error function  $w$  satisfying the boundary conditions

$$w = \frac{\partial w}{\partial n} = 0.$$

In either case given data  $z_k$  at time  $t = kh$  with  $k \geq 1$ , we look for an output-least-squares solution to the estimation problem. In the first case we consider the functional

$$J(a) = \|u_{k+1}(a) - z_{k+1}\|_Z^2 + \beta \|a\|_{H^2(\Omega)}^2$$

where the space  $Z$  is the observation space, for example,  $L^2(\Omega)$ . In the second case we consider the functional

$$J(a, u; K) = \|u - z_{k+1}\|_Z^2 + \beta \|a\|_{H^2(\Omega)}^2 + K \|\Delta w(a, u)\|_{L^2(\Omega)}^2$$

where  $K$  is taken as a penalty to force the error  $w(a, u) = 0$  at time  $t = (k+1)h$ . Thus,  $a$  is estimated at each time step. It would seem natural to initialize the minimization iteration at the current time step using the estimated coefficient from the previous time step. As information becomes available, we then refine the estimate of  $a$  by minimizing the fit-to-data functional.

By utilizing the information in this way we may obtain estimation algorithms that are faster than those using the entire history for matching. Moreover, this approach lends itself naturally to adaptive estimation of parameters. Such adaptive estimation would be appropriate for on-line identification necessary in the implementation of feed-back control strategies.

### 3. Narrow plate models.

In work with Professor D. L. Russell [5,6], we have been investigating models of structural elements that are of an intermediate character. Specifically, not only do they have a small thickness but their width is small when compared to their length as well. Such elements arise frequently in aircraft wings.

propellers, and fans blades, for example. We give the essentials of the formulation.

Let a region  $R$

$$R = \{(x, y, z): 0 \leq x \leq L, k_1(x) \leq y \leq k_2(x), \\ -\frac{1}{2} h(x, y) \leq z \leq \frac{1}{2} h(x, y)\}$$

be occupied by an elastic isotropic body satisfying the small deformation gradient assumption. We assume that the body is clamped along the line  $x = z = 0, k_1(x) \leq y \leq k_2(x)$ . We also take the functions  $k_1(\cdot)$  and  $k_2(\cdot)$  to be of the form

$$k_1(x) = p x - k(x) \text{ and } k_2(x) = p x + k(x)$$

where  $k$  is a piecewise continuous function. For our model we specify the displacement relations as a particular form of the Mindlin-Timoshenko relations where  $u, v$ , and  $w$  are the displacements in the  $x, y$ , and  $z$  directions, respectively.

$$u(x, y, z, t) = z [\Xi(x, t) + (y - px) \varphi(x, t)]$$

$$v(x, y, z, t) = z [\Psi(x, t) + (y - px) \psi(x, t)]$$

$$w(x, y, z, t) = Z(x, t) + (y - px) z(x, t) + (y - px)^2 \omega(x, t)$$

Simpler models that may be useful are obtained by omitting the quadratic term in  $(y - px)$  in the expression for  $w$  and the linear terms in  $u$  and  $v$ . These displacement assumptions give rise to the system in which  $h$  and  $k$  are constants and  $D$  is the flexural rigidity and  $G$  is the twisting modulus.

$$\begin{aligned} \frac{p k h^3}{6} \frac{\partial^2}{\partial t^2} \Xi + G k h \left( \Xi + \frac{\partial}{\partial x} Z - p z \right) - \frac{\partial}{\partial x} (2 k D \frac{\partial}{\partial x} \Xi) &= f^1 \\ (3.1) \quad \frac{p k h^3}{6} \frac{\partial^2}{\partial t^2} \Psi + G k h (\Psi + z) - \frac{\partial}{\partial x} \left( \frac{(1-\mu)}{2} k D \frac{\partial}{\partial x} \Psi \right) &= f^2 \\ 2 p k h \frac{\partial^2}{\partial t^2} Z - \frac{\partial}{\partial x} (G k h (\Xi + \frac{\partial}{\partial x} Z - p z)) &= f^3 \\ p h I_y \frac{\partial^2}{\partial t^2} z + G k h (\Psi + z - p (\Xi + \frac{\partial}{\partial x} Z - p z)) - \frac{\partial}{\partial x} (\frac{G h}{2} I_y \frac{\partial}{\partial x} z) &= f^4 \end{aligned}$$

These equations are accompanied with clamped boundary conditions at  $x = 0$  and free conditions at  $x = L$ . Clearly, by including additional terms, we can accomodate more general motions. Further, by considering a family of coupled systems of the form (3.1), it is possible to model corrugated and fan-like structures.

There are several advantages in considering the single space dimensional model (3.1) in place of two dimensional plate models.

The first is the dramatic computational speedup and savings in storage that result from replacing a 2 dimensional problem with a 1 dimensional system. Secondly, the narrow plate model incorporates the boundary behavior into the coefficients of the equations. In so doing the regularity of the solutions may be deduced much easier than in the 2-D model. The 2-D model regularity is determined by smoothness of the boundary where in the 1-D model the coefficients are obtained by integrating the functions that define the boundary.

This regularity allows us to obtain a very nice convergence theory for finite element approximations, in fact obtaining rate of convergence. The regularity theory for the 2-D model is much more complicated. While we can obtain convergence, a rate is difficult to obtain except for smooth boundaries. Such results are important since we plan to study the relation between the cross-sectional shape of the narrow plate and its the natural frequencies and mode shapes. Incidentally, results indicate that torsional frequency information is critical for determining the shape of the plate so that it is important to have a theory that accounts for more than beam-type bending.

We are also considering the use of spectral data for the identification of material parameters in narrow plate models [5]. Accordingly, we have developed a theory using a weighted least squares fit-to-data criterion that supports algorithms for the estimation of such parameters. Our theory applies whether or not the multiplicity of natural frequencies is known. Moreover, we are determining the approximation properties and stability with respect to perturbations in the spectral data. Further, we are testing our algorithm with laboratory data obtained from a particular sample for which the assumptions of the narrow plate model hold. The goal is to identify the shape of the sample. We have obtained encouraging initial results that are consistent with expected qualitative results predicted by the implicit function theorem. Investigations in this direction are continuing.

#### 4. A system of connected beams.

During this project we have been conducting an investigation concerning the estimation of coefficients or the design of beam systems [7]. In particular, we have recently been studying the following simple problem. Suppose two beams, 1 and 2, of length  $\ell_1$  and  $\ell_2$ , respectively, are joined at an angle  $\alpha$  to one another. The other end of beam 1 is clamped and the remaining end of beam 2 is free. In response to a force that is perpendicular to the plane containing the beams are deformations  $w_1$  and  $w_2$  of beams 1 and 2, respectively, and a rotation  $\theta$  of twist along beam 1. The key point for the model we have been investigating is that the angle between the beams remains fixed. For the dynamic problem with  $\alpha = 90^\circ$ , we study the following system of hyperbolic equations with coupled boundary conditions in which  $x$  and  $y$  represent local coordinates for beams 1 and 2 respectively.

$$\begin{aligned} (4.1)(i) \quad & (a w_{xx})_{xx} = f_1 \text{ in } (0, \ell_1) \\ & -(b \theta_x)_x = g \text{ in } (0, \ell_1) \\ & (c w_{2yy})_{yy} = f_2 \text{ in } (0, \ell_2) \end{aligned}$$

with boundary conditions

$$\begin{aligned} w_1(0) &= w_{1x}(0) = 0 \\ \theta(0) &= 0 \end{aligned}$$

at the clamped end,

$$\begin{aligned} (4.1)(ii) \quad & w_{1xx}(\ell_1) = 0 \\ (4.1)(iii) \quad & w_1(\ell_1) = w_2(0) \\ (4.1)(iv) \quad & \theta(\ell_1) = w_{2y}(0) \\ (4.1)(v) \quad & (b \theta_x)(\ell_1) = (c w_{2yy})(0) \\ (4.1)(vi) \quad & (a w_{1xx})_x(\ell_1) = (c w_{2yy})_y(0) \end{aligned}$$

at the junction, and

$$w_{2yy}(\ell_2) = (c w_{2yy})_y(\ell_2) = 0$$

at the free end. Equations (4.1) are associated with a potential energy functional

$$\begin{aligned} (4.2) \quad P(w_1, \theta, w_2) &= \int_0^{\ell_1} [a(x) w_{1xx}^2(x) + b(x) \theta_x^2(x)] dx + \\ &+ \int_0^{\ell_2} c(y) w_{2yy}^2(y) dy - 2 \int_0^{\ell_1} [f_1(x) w_1(x) + g(x) \theta(x)] dx - \\ &- 2 \int_0^{\ell_2} f_2(y) w_2(y) dy. \end{aligned}$$

A unique solution of the initial boundary value problem (4.1) may be shown to exist using, for example, standard Galerkin arguments with the Hilbert space



$$V = \{v = (w_1, \theta, w_2) \in H^2(0, \ell_1) \times H^1(0, \ell_1) \times H^2(0, \ell_2) : \\ w_1(0) = w_{1x}(0) = 0, \theta(0) = 0, \\ \theta(\ell_1) = w_{2y}(0), w_1(\ell_1) = w_2(0)\}.$$

To solve (4.1) numerically with a conforming method, the space of basis functions should satisfy the essential boundary conditions given in the definition of the space  $V$ . However, it is difficult to enforce the essential boundary conditions at the junction on the basis elements that are used to construct the finite dimensional approximating space, as one does, for example, with the clamped boundary conditions. We are motivated by a desire to develop and study methods that can be generalized to more complicated structures. Hence, our approach is to use basis functions that are nonconforming at the junction in the sense that they do not satisfy the essential boundary conditions at the junction. For a static problem

these conditions are enforced as constraints on the discrete version of the problem to minimize the potential energy functional and a penalty method is used to impose the conditions. For the dynamic case we consider we consider an initial boundary value problem having a potential energy functional that is obtained through a penalization of the functional (4.2).

To give a penalization formulation we define by  $W$  the space

$$W = \{v = (w_1, \theta, w_2) \in H^2(0, \ell_1) \times H^1(0, \ell_1) \times H^2(0, \ell_2) : \\ w_1(0) = w_{1x}(0) = 0, \theta(0) = 0\}.$$

and the mapping  $G: V \rightarrow \mathbb{R}^2$  by

$$G(\underline{u}) = \begin{bmatrix} w_1(\ell_1) - w_2(0) \\ \theta(\ell_1) - w_{2y}(0) \end{bmatrix}.$$

In order to obtain convergence results, apparently it is useful to define approximating problems in terms of a penalized potential energy functional to incorporate the constraint at the junction. For example, one such functional is given by

$$P_K(\underline{u}) = P(\underline{u}) + K |G(\underline{u})|^2$$

where  $\alpha \in (0,1)$ . We point out that apparently regularization is not necessary in this case as in the static case [7].

We are currently considering estimation problems for this structure based on the potential energy functional  $P_K(\cdot)$ . In this work we formulate a sequence of estimation problems where in each case the underlying system is one obtained by the penalizing the condition at the junction thereby absorbing the junction condition into a regularized functional. Hence, a sequence of estimation

problems is defined on the regularized approximating systems obtained from the penalty approximating scheme.

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- "Identifiability of discretized systems under multiple inputs," with K. Kunisch, Optimal Control Applications and Methods, 12, No 1(1991), pp 63-71.
- "Estimation of a boundary heat transfer coefficient," with K. Kunisch, Control-Theory and Advanced Technology, 7, No 1(1991), pp 55-72.
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- "Estimation of elastic parameters in coupled beam structures," to appear in J. Opt. Thy and Appl.
- "Estimation of elastic parameters in dynamic nonlinear plate models," Appl.Math. and Comp. to appear.
- "Control formulation of the initialization problem: existence, optimality conditions, and uniqueness for Burger's equation," J. Math. Anal. Appl. to appear.
- "Estimation of Material Parameters in a Nonlinear Dynamic Plate Model with Norm Constraints," submitted.
- "Continuity Properties of sets of optimal regularized output least squares estimators," submitted.
- "Formulation and Validation of Dynamical Models for Narrow Plate Motion, with D.L.Russell, submitted.
- "Identification of Parameters in Narrow Plate Models From Spectral Data, with D.L.Russell, in preparation.
- "Control of a Fluid Flow Problem with Bouyancy," in preparation.

"Time Series Identification in Plate Problems," in preparation.

"Parameter Identification in a Dynamic multiple Beam Structure," in preparation.

"Structures Modeled by Coupled Narrow Plate Equation," in preparation.

"A Moving Horizon Feedback and Feedforward Design for Vibrating System (with Finite Preview Time Via the Time-FEM), with W.N.Patten and C.C.Kuo, submitted 2nd Conference on Recent Advances in Active Control of Sound and Vibration.

"A FEM Optimal Receding Horizon Control For Systems with Previewable Inputs," with W.N.Patten and C.C. Kuo, submitted to 1993 Automatic Control Conf.

#### PARTICIPATING PROFESSIONALS

##### COLLABORATORS:

David L. Russell, Dept. of Mathematics, VPI&SU, Blacksburg, VI 24061

William N. Patten, Aerospace and Mechanical Engineering, University of Oklahoma, Norman, Oklahoma 73019

Semion Gutman, Dept. of Mathematics, University of Oklahoma, Norman, Oklahoma 73019

##### DOCTORAL STUDENTS:

Curt Holman. Topic: Identification of elastic parameters in connected beam models.

Jing Zhou. Topic: Optimal control of fluid models with bouyancy.

Zhongwu Lin. Topic: Identification and control of variational inequalities.

Ying-jun Jin

Hong Li

#### INTERACTIONS

##### MEETINGS ORGANIZED:

Year in Control Theory and Its Applications, Institute for Mathematics and Its Applications, with Profs. D.L. Russell and J.Lagnese, University of Minnesota, 1992-93.

Identification of Parameters in Distributed Systems, minisymposium, 1992 SIAM Conference on Control.

##### COLLOQUIA AND INVITED TALKS:

"Estimation of a coefficient of electrical conductivity from laboratory potential measurements," Virginia Polytechnic University and State University, May 1990.

"Estimation of a coefficient of electrical conductivity from laboratory potential measurements," University of North Carolina at Chapel Hill, May 1990.

"Continuity properties of Sets of Optimal Output Least Squares Estimators," Regional AMS Conference, University of North Texas, November 1990.

"Narrow Plate Models: Spectral Analysis, with D.L. Russell, ICIAM, July, 1991.

"Estimation of material parameters in a nonlinear plate model," AMS Workshop on PDE's, control, and Identification, Mt. Holyoke, July, 1992.

"Spectral Identification in Narrow Plate Models," with D.L. Russell, SIAM Conf. on Control and its application, Minneapolis, Sept. 1992.

"Identification of parameters in narrow plate models based on frequency data," SIAM Conf. on Control and its application, Minneapolis, Sept. 1992.